“Tricks of the Trade”

- Hard problem?
- Let’s do it the *easy* way!

Using Linearity and Time Invariance

If we can represent the input to a system as a linear combination

$$x[n] = ax_1[n] + bx_2[n]$$

then the output can be computed as

$$y[n] = ay_1[n] + by_2[n] \qquad x_1 \to y_1,\; x_2 \to y_2$$

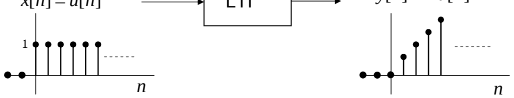
Similarly, if we can represent the input to a system as a delayed version

$x[n] = x_1[n-d]$ then the output is $y[n] = y_1[n-d]$

These two properties can be used together.

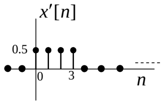
<u>example</u>

$x[n] = u[n]$ → LTI → $y[n] = n\,u[n]$

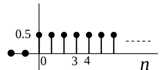


What is the response to the following input?

$x'[n]$

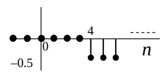


$0.5u[n]$



Write $x'[n] = 0.5u[n] - 0.5u[n-4]$

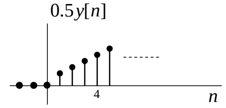
$-0.5u[n-4]$



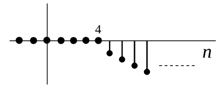
<u>example (cont’d.)</u>

The output is given by the sum:

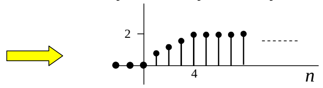
$0.5y[n]$



$-0.5y[n-4]$



$y'[n] = 0.5y[n] - 0.5y[n-4]$



Taking the Real Part

$$x[n] \longrightarrow \boxed{\begin{array}{c}\text{Real}\\ h[n]\end{array}} \longrightarrow y[n]$$

If $x[n]$ is a complex input, and $x[n] \rightarrow y[n]$

then $\mathrm{Re}\left[x[n]\right] \rightarrow \mathrm{Re}\left[y[n]\right]$

<u>proof</u>

Starting with $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

take the real part:

$$\mathrm{Re}\left[y[n]\right] = \mathrm{Re}\left[\sum_{k=-\infty}^{\infty} h[k]x[n-k]\right] = \sum_{k=-\infty}^{\infty} h[k]\,\mathrm{Re}\left[x[n-k]\right]$$

<u>example</u>

$$x[n] \longrightarrow \boxed{H(e^{j\omega}) = (2+2\cos\omega)e^{-j\omega}} \longrightarrow y[n]$$

If $x[n] = 2\cos\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$ what is $y[n]$?

<u>solution</u>

Let $z[n] = 2e^{j((\pi/3)n+\pi/4)}$ so that $x[n] = \mathrm{Re}\left\{z[n]\right\}$

The response to $z[n]$ is:

$$y_z[n] = H(e^{j\omega})\Big|_{\omega=\pi/3} \times 2e^{j((\pi/3)n+\pi/4)} = \left(3e^{-j\pi/3}\right) \times 2e^{j((\pi/3)n+\pi/4)}$$

$$= 6e^{j((\pi/3)n-\pi/12)}$$

therefore: $y[n] = \mathrm{Re}\left[y_z[n]\right] = 6\cos\left((\pi/3)n - \pi/12\right)$

<u>Exercise</u>

$$x[n] \longrightarrow \boxed{\begin{array}{c}\text{Real}\\ h[n]\end{array}} \longrightarrow y[n]$$

If $x[n]$ is a complex input, and $x[n] \rightarrow y[n]$

then show that

$$\mathrm{Im}\left[x[n]\right] \rightarrow \mathrm{Im}\left[y[n]\right]$$

and

$$x^*[n] \rightarrow y^*[n]$$ (* denotes complex conjugate)

Cascade of Systems

$$x[n] \longrightarrow \boxed{h_1[n]} \longrightarrow \boxed{h_2[n]} \longrightarrow y[n]$$

This cascade is equivalent to a single system:

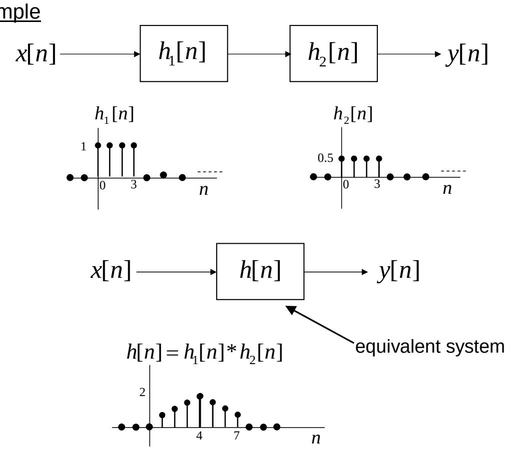
$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

where $h[n] = h_1[n] * h_2[n]$ (convolution)

<u>Corollary</u>

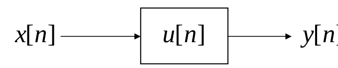
Since $h_1[n] * h_2[n] = h_2[n] * h_1[n]$ we can always interchange the order of systems in a cascade.

example



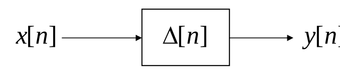
Some Special Systems

Discrete-time “integrator”



$$y[n] = x[n] * u[n] = \sum_{k=-\infty}^{\infty} x[k]u[n-k] = \sum_{k=-\infty}^{n} x[k]$$

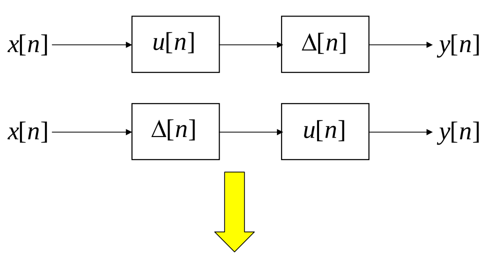
Discrete-time differencer



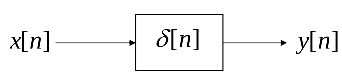
$$\Delta[n] = \delta[n] - \delta[n-1]$$

$$y[n] = x[n] - x[n-1]$$

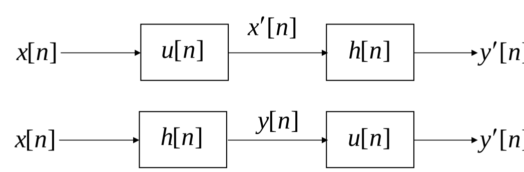
Integrator and Differencer as Inverse Systems



equivalent system



Integrating the Input



If $x[n] \to y[n]$ then $\sum_{k=-\infty}^{n} x[k] \to \sum_{k=-\infty}^{n} y[k]$

Similarly: $x[n] - x[n-1] \to y[n] - y[n-1]$